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ABSTRACT: In this work a new proposal for tool life analysis is proposed, based on the idea that it should be possible to exploit fully the data collected in tool-life experiments. An outline of Taylor equation and generalisations is discussed, underlying a possible weakness in the procedure proposed by ISO 3685 standard. After a brief review of linear multiple regression analysis, a new simple modelling of the flank wear progression, based on response surfaces depending on the logarithm of the cutting time and of the process variables is presented. Some simple equations prove that the new modelling is compatible with the Taylor equation and generalisations and a worked example from some experimental work is outlined.

KEYWORDS: tool-life testing, wear, statistical analysis
1. Introduction

Tool life data, according to ISO standard 3685 [ISO 93], are generally analysed by linear (multiple) regression. The increased availability of computing resources allows in principle straightforward calculations for a correct statistical evaluation of the data. However, from a practical standpoint, the manufacturing engineer has little help in performing tool life analysis or the relative experimental design; in fact, no specific indication of a possible procedure already in the case of multiple linear regression is specified in the mentioned standard or those relative to milling, ISO 8688 [ISO 89-1, ISO 89-2]; the same could be said in the case of recent introductory textbooks on Manufacturing at a University Degree level - which on the other hand report in some cases numerical procedures and algorithms for a simplified analysis of manufacturing processes [TLU 2000]. A modern presentation of regression methods applied to tool wear data, easy to grasp by engineers with an adequate mathematical background, should exploit matrix algebra calculations, in order to be implemented straightforwardly by commercial software available with little possibility of error or misinterpretation.

2. An outline of generalised Taylor equation and an important remark on the implicit assumption on the measurement error structure in ISO 3685.

ISO 3685 standard [ISO 93] - Tool-life testing with single-point turning tools - contains all necessary recommendations applicable, in principle, to both laboratory and production tool life testing, from the choice of machining conditions to data analysis. According to this standard, we will just briefly mention here that the adoption of a specific tool-life criteria should be adopted, for instance average flank wear equal say 0.3 mm, in order to apply the empirical Taylor equation [TAY 07]:

\[ v T_c^{-\frac{1}{k}} = C \]  \hspace{1cm} [1]

where \( v \) is the cutting speed, in m/min, \( T_c \) is the tool life, in minutes. A common remark - reported on different textbook on general manufacturing - is that the units of the left hand side are not consistent with the units of the right hand side. Namely an equation of the form:

\[ v T_c^{-\frac{1}{k}} = C \left( T_{ref}^{-\frac{1}{k}} \right) \]  \hspace{1cm} [2]

would lead to a correct dimensional presentation. However, as we refer to the specific standard and with the aim of aiding an easy interpretation of tool-life data
for practical purposes, we will continue to use the nomenclature expressed by equation 1. It is customary - but we already observe also delicate - to plot and analyse in a log log scale the Taylor equation, transforming in a linear relationship the non-linear power law expressed by equation 1. Different authors have presented in along the years extensions of the Taylor equation; Shaw [SHA 96] reports for instance an equation of the form:

$$T_c v_c^{1/n} f^{1/m} a_p^{1/l} = C^n$$  \[3\]

Where f is the feed and a_p is the depth of cut and n,m,l generalise the Taylor exponent. The logarithmic transformation may be also applied to the generalised Taylor equations leading to different formulations of the tool life equation. A recent review of possible equations which has been presented in a paper by Cukor and Kuljanic [CUK 99], discusses some 8 different tool life equations, presenting them already in the form of a relationship between the logarithm of the tool life y and some, possibly simple, function of several variables which are called:

$$x_1 = \ln v_c; \quad x_2 = \ln f; \quad x_3 = \ln a_p; \quad x_4 = \ln h_e;$$

where he is the equivalent chip thickness. The equations all have a form:

$$y = \varphi(x_1, x_2, x_3, x_4)$$  \[4\]

where y= ln Tc. The same equation could be presented in a compact vectorial form:

$$y = \varphi(x)$$  \[5\]

where now x stands for a row vector of one or more of the variables x_1, x_2, x_3, x_4 presented above. Referring to equation 5, a tool life experiment could generally be presented as a number n of observations, that one could write for every i observation:

$$y_i = \varphi(x_i) + \varepsilon_i$$  \[6\]

with $E(\varepsilon_i) = 0; E(\varepsilon_i \varepsilon_j) = 0$ for $i \neq j; \ E(\varepsilon_i^2) = 0$.

Where we have introduced the unobservable error $\varepsilon_i$ that is unavoidable in any measuring, and used the expression $E(\ )$ as the standard statistical shorthand notation for the mean of a random variable. We should note that we have transformed actual physical variables to their logarithms. In this sense, we should be
careful, as with this operation we inherently assume that the measurement error is multiplicative [DRA 98] in its nature, as it affects the measured variable - in this case the tool life - in such a way that, with the definition of a "real" error free tool life time:

\[ T_e = \hat{T_e} \cdot Error \]  
\[ y = \ln T_e + \ln (Error) = \ln \hat{T_e} + \epsilon \]  

The approach that has been used in technical standards, noticeably in ISO 3685, assumes implicitly that errors add up additively in the log log relationships; if that would not be assumed for sure, one could not present results and additional statistics for the estimation of confidence intervals, as reported in the ISO 3685 annex F, point F5. We have then to turn to the evaluation of the tool life time, \( T_C \). We will discuss for the sake of simplicity, and also because it is the most widespread way of industrial tool life testing, the case of the flank wear criterion. So, assigned the tool wear criterion, the tool life is obtained from experimental plots which report monitoring of the tool wear progression, called in this case development of flank wear, reported usually at discrete times. If we would like to keep unaltered the analysis and the conclusions of the ISO 3685 standard we must employ an empirical model of the flank wear progression which relates flank wear to cutting time with an equation of the type:

\[ v_B = m \ln T + q \]  

Where \( m \) and \( q \) are just empirical coefficients for modelling the linear trend. In such a way, this relationship between \( v_B \) and \( T \) transforms additive uncertainty on \( v_B \) on additive uncertainty and finally to multiplicative uncertainty on \( \ln T \), as required. With the current way of developing tool-life analysis, the only way to avoid such assumption, which on a practical ground seems in some different cases suitable, is to apply directly non-linear regression analysis to equations of the form of equation 3. In this case linear analysis could be adopted for initial estimates of the points, but its results could not be employed for interval confidence estimation.

3. A review of related matrix approach in linear multiple regression [MON 92, MON 94]

We shall report briefly a review of statistical computations that may lead to some simple computer aided calculations. There exists many different excellent commercial software able to perform routinely statistical calculations, but we believe that there must be a way between the manual calculation of all data needed, as ISO 3685, and entering data in sophisticated statistical program. The matrix approach to linear multiple regression has been introduced since several decades in
basic statistical procedures, but only in recent years has become easy to implement in basic commercial software. Recalling equation 4, we could write a linear model:

\[ y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \varepsilon_i = \beta_0 + \sum_{j=1}^{n} \beta_j x_{ij} + \varepsilon_i \quad [9] \]

or in a more compact vectorial form:

\[ \mathbf{y} = \mathbf{X}\beta + \varepsilon \quad [9'] \]

where

\[ \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1k} \\ 1 & x_{21} & \cdots & x_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{nk} \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} \]

\(k\) could assume values from 1 (considering only the cutting speed responsible of the tool wear) to 4, that is, including cutting speed, feed, depth of cut and average chip thickness as in equation 4; this in the case we believe there are no interactions between the various independent process parameter factors - actually their logarithms. If we believe it is possible to introduce a more complex model, we could add up to the model in principle any combination of the \(x\) variables up to the power we like, and still use the same formalisms we are describing here. How many factors or simply which model to choose is not generally a simple problem, as one should use specific techniques in a model building framework, using for instance variable selection techniques. On a practical ground however, it is unlikely that a second order full model would be used, for the necessary amount of data which one experimenter usually is not able to provide. The great advantage of matrix approach, as reported in general textbooks on linear regression, or even on intermediate textbooks of statistics for engineers stands on the simple expression - and computation - of the least square estimator \(\hat{\beta}\), that may be expressed as:

\[ \hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \quad [10] \]

where the symbol ' represents matrix transpose.
The fitted observations vector may be evaluated as:

$$\hat{y} = X\hat{\beta} = X(X'X)^{-1}X'y$$  \[11\]

The estimated residual error vector $e$ is then:

$$e = y - \hat{y}$$  \[12\]

It is then straightforward to report the residual sum of squares, customarily called $SSE$ (sum of squared errors):

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = e' e$$  \[13\]

The residual mean square $MSE$ or briefly $s^2$, is a model dependent estimate of the variance of the regression, $\sigma$.

$$s^2 = MSE = \frac{SSE}{n - (k + 1)} = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n - (k + 1)} = \frac{e'e}{n - (k + 1)}$$  \[14\]

What is of central interest is the preliminary testing of the significance of the whole regression model employed. In multiple regression analysis it is customary to employ a table reporting the analysis of the variance for the significance of regression with normality assumptions:

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>$F_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>SSR</td>
<td>$K$</td>
<td>MSR=$SSR/k$</td>
<td>MSR/MSE</td>
</tr>
<tr>
<td>Residual</td>
<td>SSE</td>
<td>$n-k-1$</td>
<td>$MSE=SSE/(n-k-1)$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$Syy$</td>
<td>$n-1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Customary Analysis of Variance for Multiple Regression

This is basically the same table reported at page 39 of the ISO standard. We should accept the regression model if the following inequality holds:

$$F_0 = \frac{MSR}{MSE} < f(\alpha, k, n-k-1) \iff p = F(F_0, k, n-k-1) < \alpha$$
Where $F$ is the Fisher distribution, $f$ its inverse and $\alpha$ the value which gives a confidence level of 100 $(1-\alpha)$. ISO 3685, even not reporting directly $F$ distribution, uses 95% confidence level for the t-test described below, i.e. adopts $\alpha=0.05$; the Table F2 is probably to improve, as it would be useful to mention explicitly such a $\alpha$ value. Regarding the $F$ distribution and its inverse, such a function is available now not only tabulated in statistical textbooks, but also in commercial basic spreadsheets for office applications. Again, with the concerns reported on the error structure - here and implicitly in ISO 3685 we assume it distributed normally on the log $T$ data - after validating the regression model we can estimate the confidence intervals on the regression coefficients. We should define a matrix $C$:

$$C = \left(X'X\right)^{-1} \quad [15]$$

In order to compute the confidence intervals on the regression coefficients, - which are again linear combinations of the observations - the following intervals hold, under normality:

$$\hat{\beta}_j - \frac{t_{\alpha/2, n-(k+1)}}{s \cdot \sqrt{C_{jj}}} \leq \beta_j \leq \hat{\beta}_j + \frac{t_{\alpha/2, n-(k+1)}}{s \cdot \sqrt{C_{jj}}} \quad [16]$$

Consequently, a $t_0$ may be defined and the relative $P$ value as we have done for $F$:

$$t_0 = \frac{\hat{\beta}_j}{s \cdot \sqrt{C_{jj}}} = \frac{\hat{\beta}_j}{se(\hat{\beta}_j)} \iff P = t(t_0, n - k - 1) < \alpha \quad [16']$$

A confidence interval for the mean response may be estimated with a similar procedure, defining a vector $x_0$:

$$x_0' = \begin{bmatrix} x_{01} & x_{02} & \cdots & x_{0n} \end{bmatrix}$$

The evaluated value is:

$$\hat{y}_0 = x_0' \hat{\beta} \quad [17]$$

and consequently:

$$\hat{y}_0 - \frac{t_{\alpha/2, n-(k+1)}}{s \cdot \sqrt{x_0'Cx_0}} \leq E(y|x_0) \leq \hat{y}_0 + \frac{t_{\alpha/2, n-(k+1)}}{s \cdot \sqrt{x_0'Cx_0}} \quad [18]$$
4. Exploiting fully the data on the flank wear progression: a simple response surface model

When we carry out a tool-life experiment, we have some additional information that in our opinion could be fully exploited, with only some additional computational cost but with the significant advantage that no implicit assumption(s) should be made for what concerns the error distribution in the tool-life. Here we should consider that the quantitative expression of wear, let’s call it \( v_B \) as the flank wear, could be considered a function of time, cutting speed, feed, and all the other variables, like equation 5; looking at the time dependence - with fixed cutting parameters - the flank wear may be modelled with simple smooth functions of time, but up to date no specific general property has been attributed. Guided by the suggestion proposed by equation 8, it could be interesting to consider the flank wear as a function of

\[
x_1 = \ln v_c; \quad x_2 = \ln f; \quad x_3 = \ln a_p; \quad x_4 = \ln h_c; \quad y = \ln T
\]

\[ v_B = f(x_1, x_2, x_3, x_4, y) \]  \[ \text{[19]} \]

In such a case, if we would model the \( f \) function with an (hyper)plane, i.e. a multiple linear function, we would derive immediately Taylor’s original equation or some of its generalisations. For the sake of simplicity and for presenting in graphical form our findings, we shall refer now with a simple situation, frequently met in industrial situations, the one in which the only variable is the cutting speed \( v \); we shall call \( \ln v = x \), neglecting the 1 subscript starting from this point. Modelling the response surface of \( v_B \) with a second order polynomial we would obtain:

\[
v_B = ax^2 + by^2 + cxy + dx + ey + f \]  \[ \text{[20]} \]

Here, as we have noted, if we pose \( a=0, b=, c=0 \) (linear model) we obtain:

\[
v_B = dx + ey + f \]  \[ \text{[21]} \]

i.e. Taylor tool-life equation in its logarithmic form:

\[
\frac{1}{k} \ln(vT^{-k}) = \ln C \quad \leftrightarrow \quad \ln(v) - \frac{1}{k} \ln(T) = \ln C
\]

\[
\leftrightarrow x - \frac{1}{k} y = \ln C \]  \[ \text{[22]} \]
and by comparing equation 22 with a rewritten form of equation 21:

\[
x + \frac{e}{d} y + \frac{f}{d} = \frac{v_B}{d}
\]

we obtain:

\[
-\frac{1}{k} = \frac{e}{d}; \quad \ln C = \frac{v_B}{d} - \frac{f}{d}
\]

This model would predict a k value in ISO 3685 equation not dependent on the specific value of the flank wear imposed, that may be acceptable on a practical ground if in the time scales we consider we have the same (combination of) wear mechanisms in the tool life experiments. As for what concerns the analysis of equation 19 or its simplified version 20, we can now carry out the computation of statistical confidence, without any restriction, as \(v_B\) is a linear length measure, for which a gaussian distribution may be easily accepted. The procedure that allows to evaluate the coefficients is the same employed in paragraph 3, provided now that the single columns of the matrix X are \(x_i\) functions. For instance, in the case of equation 20 is applied, matrix X becomes:

\[
y = \begin{bmatrix} vB_1 \\ vB_2 \\ \vdots \\ vB_n \end{bmatrix}, \quad X = \begin{bmatrix} 1 & x_1^2 & \cdots & y_1 \\ 1 & x_2^2 & \cdots & y_2 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n^2 & \cdots & y_n \end{bmatrix}, \quad \beta = \begin{bmatrix} f \\ a \\ b \\ c \\ d \\ e \end{bmatrix}
\]

5. A simple example in tool-life testing

As an example, we employ the experimental results of tool-life testing of high speed steel end mills, that were tested in order to optimise a Physical Vapour Deposition process (PVD) for what concerns temperature of deposition and coating thickness. A number of 27 different tool-life experiments were performed, we shall consider here just a set of sub experiments in which several H.S.S. (8 % Co, DIN S2-10-1-8) milling cutters with diameter D=10 mm., Z=4 teeth were coated in a
PVD process with a coating of 3 \( \mu \)m., at a temperature of deposition of 430 °C. Cutting conditions were: a fixed feed for tooth \( s_z=0.042 \) mm/round, \( v \) varying at three levels: 18.85, 28.27, 37.7 m/min, depth of cut \( a_p=10 \) mm., slot milling \( (a_r=D) \), refrigerant, a 5% emulsion of Sinol X% MR. A machining centre Kitamura, model Mycenter H 400 Supercell, with spindle speed variable from 60 to 6000 rpm, and an available power of 11 kW was employed. The workpiece material was steel 38NiCrMo5, hardness 300 HV. Steel composition in weight percent was:

<table>
<thead>
<tr>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>P</th>
<th>S</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.39</td>
<td>0.60</td>
<td>0.26</td>
<td>0.030</td>
<td>0.070</td>
<td>0.80</td>
<td>0.73</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 2. Steel composition in weight percent for the cutting test.

At first an average flank wear criterion of 0.3 mm was employed, that was measured with a Pico Scopeman digital microscope. In any scale one would adopt analysis, an experienced eye would note that the value for the sub Test 16, speed \( v=18.85 \) for the value of time \( T=100 \) min is probably an experimental error, that may happen quite frequently in cutting tests.

<table>
<thead>
<tr>
<th>( v=18.85 ) m/min (sub Test 16)</th>
<th>( v=28.27 ) m/min (sub Test 6)</th>
<th>( v=28.27 ) m/min (sub Test 7)</th>
<th>( v=37.7 ) m/min (sub Test 20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time, min</td>
<td>vb, mm</td>
<td>Time, min</td>
<td>vb, mm</td>
</tr>
<tr>
<td>10</td>
<td>0.16</td>
<td>6.67</td>
<td>0.18</td>
</tr>
<tr>
<td>20</td>
<td>0.18</td>
<td>13.33</td>
<td>0.2</td>
</tr>
<tr>
<td>30</td>
<td>0.19</td>
<td>20</td>
<td>0.21</td>
</tr>
<tr>
<td>40</td>
<td>0.2</td>
<td>26.67</td>
<td>0.24</td>
</tr>
<tr>
<td>50</td>
<td>0.21</td>
<td>33.33</td>
<td>0.26</td>
</tr>
<tr>
<td>60</td>
<td>0.22</td>
<td>40</td>
<td>0.28</td>
</tr>
<tr>
<td>70</td>
<td>0.24</td>
<td>46.67</td>
<td>0.31</td>
</tr>
<tr>
<td>80</td>
<td>0.25</td>
<td></td>
<td>53.3</td>
</tr>
<tr>
<td>90</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Tool wear data for the experimental example.

Next figures 1A and 1B report tool wear progression in two different scales, the one suggested by figure 9 of ISO 3685 standard, which suggests to employ at least 5 flank wear versus time points for each cutting speed. The resulting linear regressions respectively for the linear time - linear flank wear (Fig.1A) evolution, are reported in Table 4A expressed in a form \( y=mx+q \); the probability level \( P \) - value the 100 (1-P) confidence, are reported in the same Table. The corresponding values for the and logarithmic time - linear flank wear evolution are reported in Table 4B.
Figure 1A: tool wear progression, linear scale. Triangles, $v=37.7$ m/min, Crosses $v=28.27$ m/min, Circles $v=28.27$ m/min, Lines $v=18.85$ m/min.

Figure 1B: tool wear progression, with time on a logarithmic scale. Triangles, $v=37.7$ m/min, Crosses $v=28.27$ m/min, Circles $v=28.27$ m/min, Lines $v=18.85$. 
The logarithmic fit is in nearly all cases slightly worse than the linear fit, but apart from the test 16 - which has the recalled suspect (100, 0.31) point - the two trends are comparable.

<table>
<thead>
<tr>
<th></th>
<th>v=18.85 m/min (sub Test 16)</th>
<th>v=28.27 m/min (sub Test 6)</th>
<th>v=28.27 m/min (sub Test 7)</th>
<th>v=37.7 m/min (sub Test 20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>m, mm/min²</td>
<td>0.145333</td>
<td>0.15428511</td>
<td>0.168547</td>
<td>0.22</td>
</tr>
<tr>
<td>q, mm/min</td>
<td>0.001376</td>
<td>0.00321425</td>
<td>0.00259</td>
<td>0.002571</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9245</td>
<td>0.98902</td>
<td>0.8947</td>
<td>0.9806</td>
</tr>
<tr>
<td>P - value</td>
<td>0.0056</td>
<td>0.026</td>
<td>0.018</td>
<td>0.047</td>
</tr>
</tbody>
</table>

**Table 4A: Constants for the flank wear - time regression**

<table>
<thead>
<tr>
<th></th>
<th>v=18.85 m/min (sub Test 16)</th>
<th>v=28.27 m/min (sub Test 6)</th>
<th>v=28.27 m/min (sub Test 7)</th>
<th>v=37.7 m/min (sub Test 20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>0.020062</td>
<td>0.04136534</td>
<td>0.043153</td>
<td>0.168993</td>
</tr>
<tr>
<td>Q</td>
<td>0.052698</td>
<td>0.06376613</td>
<td>0.063022</td>
<td>0.035479</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.7951</td>
<td>0.8798</td>
<td>0.9829</td>
<td>0.9368</td>
</tr>
<tr>
<td>P - value</td>
<td>0.0081</td>
<td>0.032</td>
<td>0.015</td>
<td>0.050</td>
</tr>
</tbody>
</table>

**Table 4B: Constants for the flank wear - logarithm (time) regression**

As ISO 3685 suggests, we will avoid to extrapolate values, and consider the values of the tool life for $v_B=0.25$ mm., which are reported in Table 5:

<table>
<thead>
<tr>
<th></th>
<th>v=18.85 m/min (sub Test 16)</th>
<th>v=28.27 m/min (sub Test 6)</th>
<th>v=28.27 m/min (sub Test 7)</th>
<th>v=37.7 m/min (sub Test 20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tc, min linear - linear flank wear modelling</td>
<td>76.1</td>
<td>29.8</td>
<td>31.4</td>
<td>11.7</td>
</tr>
<tr>
<td>Tc, min linear - logarithmic flank wear modelling</td>
<td>78.5</td>
<td>26.4</td>
<td>26.6</td>
<td>9.8</td>
</tr>
</tbody>
</table>

**Table 5. Tool life for $v_B=0.25$ mm., according to the two different of flank wear**

For the two different data sets, regression analysis gives the values for k, C, Taylor equation constants and $R^2$ and P-value, regression outputs, which are reported in Table 6. Even if the P and $R^2$ are comparable, we note that - as for the hypotheses with have discussed on the error structure - we could strictly consider just the former results. So relative terms are bracketed.
Table 6. Taylor equation values, according to ISO 3685 and to the two different tool wear progression models. First row terms are bracketed as the error structure could not be assumed as normal.

In the case of linear -logarithmic trend we can build the analysis of variance (ANOVA) table for the regression that was presented in general form in Table 1, that is presented in the following Table 7.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>2.149942</td>
<td>1</td>
<td>2.149942</td>
<td>255.09</td>
</tr>
<tr>
<td>Residual</td>
<td>0.016856</td>
<td>2</td>
<td>0.008428</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2.166798</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Analysis of variance for the linear - logarithmic flank wear modelling

Modelling all the flank wear data, according to the new approach in the tool wear progression presented in simplified form in equation 21 leads to an equation of the form:

\[ v_B = 0.149x + 0.0549y - 0.427[m m] \]  

which allows to calculate \( k = -2.71 \) and \( C = 94.00 \), according to equation 23. These values are intermediate values between the values reported in Table 6. Also for this model it is possible to make the ANOVA analysis, which is presented in the following Table 8. The F0 value corresponds to a P - value very small (less than \( 10^{-12} \)), and may be considered much more satisfying than the direct Taylor equation regression. The reason stands in the fact that we have many more data to fit, while on usual analysis we discard all the information we have gained from the tool-wear progression, summarising it in just one number, the tool-life. As for the coefficients all the three coefficients of the model are significant, as would be expected by a relationship totally compatible with the Taylor equation.
Table 8. Analysis of Variance for the model of flank wear progression presented in equation 21

The procedure to test the significance of single coefficients exploits eqn. 16' and is:

<table>
<thead>
<tr>
<th>coefficient</th>
<th>d</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>P- Value</td>
<td>3.08389E-13</td>
<td>3.08389E-13</td>
<td>5.22448E-09</td>
</tr>
</tbody>
</table>

Table 9. Probability P- Values for the coefficients of equation 24

Figure 2. Modelling of the flank wear progression with a linear model in $x=\ln T$ and $y=\ln v$, according to equation 21; dots represent experimental points.
Finally, we can consider a second order model of the flank wear progression in $\ln v$ and $\ln T$, as presented in equation 20. The computed model is:

$$v_B = -0.0121x^2 - 0.00338y^2 - 0.0290xy + 0.3264x + 0.1717y - 0.91424[\text{mm}]$$

The relative ANOVA table is presented in Table 9:

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>$F_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>0.050028676</td>
<td>5</td>
<td>0.0100057</td>
<td>55.98</td>
</tr>
<tr>
<td>Residual</td>
<td>0.004468099</td>
<td>25</td>
<td>0.0001787</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.054496774</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9. Analysis of Variance for the model of flank wear progression presented in equation 20.

The computed $P$-Value is of the order of less than $10^{-12}$, again very satisfactorily, looking at the ISO 3685 values. However, in this case, not all the coefficients are significant, specifically the coefficients $a$ and $b$, which exceed by far an acceptable level of 0.05. This means that a simpler model could be more significant. The relative $P$-values are:

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.238908</td>
</tr>
<tr>
<td>b</td>
<td>0.001708</td>
</tr>
<tr>
<td>c</td>
<td>0.034897</td>
</tr>
<tr>
<td>d</td>
<td>0.297512</td>
</tr>
<tr>
<td>e</td>
<td>0.031957</td>
</tr>
<tr>
<td>f</td>
<td>0.226652</td>
</tr>
</tbody>
</table>

Table 10. $P$-values for the full second order polynomial according to equation 20.

The application of a backward elimination algorithm on a commercial statistical package leads to a model in which the contribution of $a$ and $d$ is neglected, with an ANOVA Table:

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>$F_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>SSR</td>
<td>3</td>
<td>0.0165554</td>
<td>92.53</td>
</tr>
<tr>
<td>Residual</td>
<td>SSE</td>
<td>27</td>
<td>0.0001789</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>Syy</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10. Analysis of Variance for the model of flank wear progression presented in equation 20 with coefficients $a$ and $d=0$ - after backward elimination.
All the coefficients are significant with the P - values as following:

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>b</th>
<th>c</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coef. Value</td>
<td>0.02171222</td>
<td>0.05086313</td>
<td>-0.2435718</td>
<td>0.03640418</td>
</tr>
<tr>
<td>P - value</td>
<td>1.1E-05</td>
<td>1.17E-13</td>
<td>2.64E-08</td>
<td>1.89E-07</td>
</tr>
</tbody>
</table>

Table 11. P- coefficients for the model of flank wear progression presented in equation 20 with coefficients a and d=0 - after backward elimination.

Now coming back to the general form of the response surface, the fitted equation for $v_B$ may be used for calculating values for tool life at any specified $v_B$ values, within the experimental ranges considered. We can rewrite eqn.20 for a given value of $x=\ln v$:

$$by^2 + y(cx + e) + ax^2 + dx + f - v_B = 0 \quad [25]$$

that solved leads to two roots, the positive one $y_1$ is the one we can use:

$$y_1 = \frac{1}{2b} \left( -cx - e + \sqrt{c^2x^2 + 2cxe + e^2 - 4bdx - 4bf + 4bv_B - 4bax^2} \right) \quad [26]$$

In such a way, it is possible to calculate the tool life directly, once that the value of cutting speed and the tool wear assigned are given.

6. Conclusions

The new approach may be interesting not only in the specific activity of tool life testing, but for providing a quantitative expression for tool life in complex machining operations. The possibility to link directly tool wear progression to cutting parameters and time directly, fully exploiting the data collected along the test for tool life determination appears potentially interesting, as this could help in gaining more information, without time consuming and expensive additional machining tests. In this sense this work has the same aim of the studies performed to reduce the number of necessary tests in tool-life testing [KUL 80, CUK 99]. While there are other contemporary techniques (cfr. for instance [DIM 2000-1, DIM 200-1) to tool-life modelling which are of interest but require specific software, it is important to remark that computationally the approach presented here is no more demanding than standard calculations for tool-life testing, like the ones presented in ISO 3685; in fact, all the necessary calculations here may be carried out not only on specialised software packages for statistical applications, but on commercial spreadsheets as well. This could help the applicability of a possibly improved approach, even in industrial conditions; for this reason further work is in progress to test the performance of this methodology in different experimental applications.
References


[TAY 07] TAYLOR, F. W., On the Art of Cutting Metals, Transactions of ASME, 28, 31-279, 1907