A Method for Computation and Assessment of NC Tool-paths in Free-form Curve Format

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ABSTRACT :The paper deals with tool-path calculation in B-spline curve format. We expose the geometric constraints necessary to correctly define a tool-path. The association problem is then developed. Assessment methods are suggested that allows the validation of our calculation method. In particular, the interest of the partitioning of the tool-path is clearly showed.

KEY WORDS :CAM, Numerical Control, Precision, Free form machining

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1. Introduction

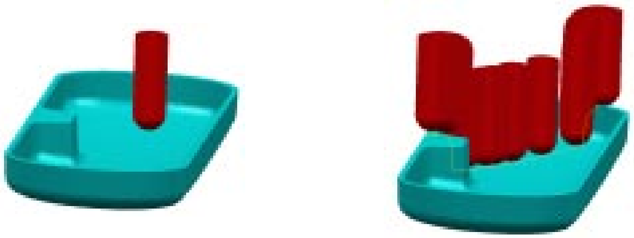
1.1. *High Speed machining of free-forms*

For a few years, the elaboration process of free-form surfaces has undergone strong evolutions [DUR 99]. For instance, the use of CAD/CAM and High Speed Milling allows the decrease in machining time and in polishing or rectifying time [HOC 97], [SCH 95], [TUS 93]. As a result, the final machined shape corresponds to the envelope of the tool movements.

From now on, the CAD activity defines a virtual model of the part to be carried out. Therefore, the elaboration process must be able to produce parts accurate to the virtual model. Moreover, the machined part must be in accordance with the geometrical specifications. Unfortunately, deviations between the virtual model and the machined form appear all process long [DUC 98a]. Among all these deviations, we consider in particular those produced by the calculation of the tool-path. If the calculated tool-path does not meet the geometrical specifications, then the machined part cannot be correct. Moreover if the tool-path does not simplify the treatment by the Numerical Controller (NC), then the NC can slow down the machining speed or can disturb the continuity of the tool movement. As a result, the machining time is increased and the surface quality is decreased.

1.2. *Methods of tool-path calculation*

Methods of tool-path calculation can be divided in four steps: calculation of one tool location, detection of the interference, calculation of one single path, and calculation of the whole tool-path [DUC 98b]. Most of the methods suggested in literature are well adapted for the linear format. In this case, the tool-path consists of a succession of line segments [DRA 97],[HWA 92], [JEN 96].



1) calculation of one tool location *3) calculation of a single path*

2) discontinuity detection *4) the whole tool-path*

Figure 1. *tool-path calculation*

Concerning the first step, the calculation of one tool location consists in the determination of the location of one fixed point of the tool, relatively to the surface to be machined. The main calculation methods rely on the use of the offset surface [KIM 95], or the inverse offset method [SUZ 91], or associated to the Z-buffer method [SAI 91].

The detection of the discontinuities allows determining the tool location at the linking between surfaces, where discontinuities in position, tangency or curvature may exist. This allows the definition of an interference free tool path [CHO 94].

Then, the single path, which corresponds to a succession of tool locations calculated according the considered machining strategy, is obtained. The number of tool locations is strongly linked to the machining tolerance, the value of which is defined according to the geometrical specifications [KIM 95], [DUC 98b].

At final, the whole tool path is obtained by calculation of successive single paths. The distance between two successive single paths must respect the maximal scallop height allowed [LIN 96], [LON 87].

Therefore, the calculated tool path consists in a set of points which is transmitted to the NC. The NC makes the linear interpolation between two successive points. This description format of the tool-path reaches its limits in HSM. Despite the fact that linear segments give poor information on the free-form to be machined, the respect of the expected quality on the free-form requires a large number of points (that means a large size of the corresponding files). Therefore, the treatment of information may be slowed down. Moreover, dynamical problems may occur due to the small size of elementary segments between two points.

All this factors represent limits in the process performance, and consequently, in the quality of the part surface. Then, it seems necessary to focus on the improvement of the communication between the CAD system and the NC through the use of other description formats of the tool-path. The aim of this paper is to analyse various calculation methods of the machining tool-path in a free-form curve format. Moreover, the performance assessment of the free-form curve format is performed that shows the relevance of such a description format for the machining of free-form surfaces.

2. The free-form curve format for the tool path description

Since 1996, the elaboration process of free-form surfaces has evolved so as to integrate new description formats of the machining tool path. For instance, recent Numerical Controllers are now able to read and to interpret tool paths defined in free-form curve formats: B-spline, Nurbs, polynomial, … Thus, the problem is to be able to calculate the tool path in such a format with respect to the geometrical specifications.

2.1. *The issue*

First, let us remind that the calculation of a tool path consists in the transformation of the surface geometry into *tool path geomet*ry. The description format of such geometry must be understandable by the Numerical Controller. The transformation corresponds to a real modelling of the geometry. This modelling is carried out taking into account the tool geometry and the surface geometry, and through the calculation of the location of a tool point. This tool point location is calculated from a contact point between the tool and the surface. As the tool path geometry is generally a discrete set of points, two steps of discretization are required to specify the maximum distance allowed between the tool path and the initial surface geometry. One of these discretization steps is the machining tolerance. If we consider the ideal tool-path that machines the surface exactly, the machining tolerance specifies the envelope in which the tool path is located relatively to the ideal tool-path. Therefore, the real calculated tool path is a modelling of the ideal tool path. Note that the equation of the ideal tool-path is generally unknown.

Obviously, the tool-path geometry must be in accordance with the description formats interpretable by the NC. Although, the geometry transformation does not cause problems with the linear format, it is not the case with the free-form curve format. The most employed curve models are the canonical model, the Bezier model and the B-spline model [SIE 98]. Among these models, the cubic B-spline appears to be one of the most interesting for it ensures a C^2 continuity while limiting oscillations and the size of data files. Our developments are based on this model.

To conclude, the issue is then to model the ideal tool path with a free-form curve, which description is held up by the numerical controller, with respect to the machining tolerance. As the resulting curve must be representative of a tool path, geometric constraints must be defined to that end. Therefore, this leads to an interpolation problem under constraints.

2.2. *Geometric constraints associated to the tool path*

As said in the previous section the first and foremost constraint is the machining tolerance. Thus, the calculated curve must be included into a pipe which spine is the ideal tool-path curve, and the radius of which is the tolerance value. In other words, the distance between the calculated curve and the ideal curve must be less than the machining tolerance for each point of the curve.

As the whole surface can not be approximated or machined by a unique curve, another main constraint concerns the linking between two calculated curves. Obviously, both curve extremities must lie on the ideal curve so that the linking between two calculated curves is at least C^0 continuous.

The last constraint is linked to the dynamical behavior of the curve. The treatment by the numerical controller is simplified when the tool always covers the same trajectory length for a given time. To satisfy this constraint, the parameterisation associated to the curve must be equal to the chord length.

If the calculated curve respects all these constraints, it can be considered as a tool path. However, this is not sufficient to define satisfactory machined forms. Thus, we suggest two additional constraints in order to define the form of the curve into the pipe and at the extremities (figure 2).

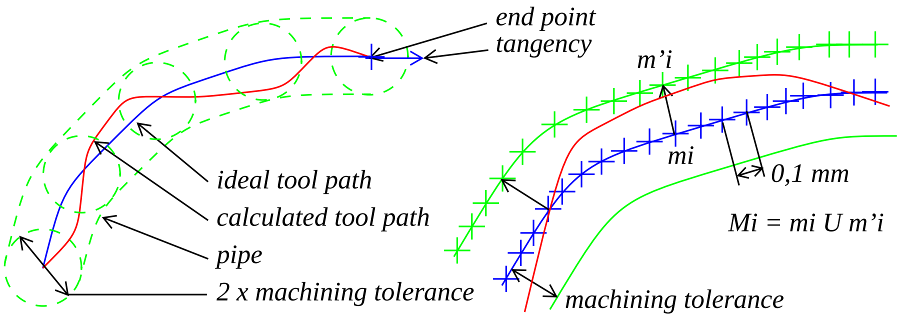


Figure 2. *Pipe definition and samplings of the curves*

Concerning this last point, the additional constraint we suggest is to impose C^1 continuity between curves by specifying the tangency direction at the extremities. The choice of C^1 continuity instead of C^2 continuity is justified. Indeed, the C^2 continuity involves that the shape of the curve into the pipe is more constrained. It then becomes difficult to hold on the curve into the pipe.

Between the extremities, the sole respect the machining tolerance of the curve may lead to waves of the calculated curve around the ideal curve. In particular, these waves are due to various factors such as:

-a high curve degree,

-a great value of the curvature radius in particular points,

-a low machining tolerance,

-a deviation between the imposed tangencies and those of the ideal curve.

The magnitude of these waves is less than the machining tolerance, and the wavelength is equal to the distance between two interpolating points. Therefore, it is advisable to avoid waves during the building of the curves.

2.3. *Geometry of the ideal tool path curve*

The calculation of the tool path curve requires a good definition of the ideal tool path curve. The definition by only a few points is not sufficient for the calculated curve constitutes the guidance curve of the tool onto the surface. Thus, the building of the curve is not a common association problem.

Constraints on the ideal tool path curve are directly linked to the constraints that define the tool path. For instance, to respect the inclusion of the whole tool-path in pipes which are very small or when the curve presents C^2 discontinuity, local point densification must be performed. In this case, the solution consists in the partitioning of the ideal tool-path curve into C^2 continuous portions. This solution is preferred to the local densification of the control points near the discontinuity, for it allows, in particular, the limitation of local waves and moreover, the decrease in calculation time.

The other main constraint is the calculation of the curve length of the ideal curve. This parameter is important for the respect of the dynamical constraint. Here, the calculation of the curve length must be as accurate as possible, for the accuracy of the calculation is directly linked to the accuracy of the association problem. If the calculated length is less than the exact value, then the calculated curve is smaller than the ideal curve and may not be included into the pipe (figure 2). On the opposite, if the value is too large, then the calculated curve is greater and may oscillate into the pipe. Tests showed that when the deviation is of 0.1mm for a curve length of 100 mm, the calculated curve could not be included into a pipe which radius is 0.01mm unless points were added.

The last point concerns the precision of the calculation of each point of the ideal tool path curve. Each deviation onto the calculated point is of influence onto the calculated tool-path, for it may strongly alter the association problem.

To conclude, the calculation of the tool-path as a free-form curve requires a modeling of the ideal curve very accurate and complete. The modelling of the surface through a meshing can lead to less accurate tool paths.

2.4. *Criteria for the performance assessment*

To assess the performance of the free-form curve format, it is necessary to define evaluation criteria. These criteria must assess the performance of the calculated tool-path, but must also assess the quality of the machined free-form surface [DUC 00].

Concerning the calculated tool path, it must respect constraints previously exposed:

-the machining tolerance,

-the C^0 continuity of the linking between two curves,
-the dynamical constraint,
-the limitation of the waves,
-the tangency constraints at the extremities.

To evaluate the respect of all these constraints, methods have been developed, relying on the calculation of the distance between the tool path and the initial surface [DUC 00].

The final free-form machined with the calculated tool-path must respect the following geometrical specifications:

-form deviation,
-characteristic lines of the free-form,
-convexity direction.

Moreover, the machining on the machine tool must lead to a final form free from any marks (that can be due to a wrong dynamical behavior of the machine tool). At last, the real feedrate of the tool must be as close as possible to the selected feedrate.

Concerning the respect of all the geometric specifications, only the form deviation can be quantified. Other criteria are evaluated in a qualitative way.

3. Calculation of the tool path in free-form curve format

In this section we present the calculation methods of the tool path we developed. The initial method developed by [DUC 96] has been expanded so that it may integrate additional tangency constraints. Moreover, we test the interest of smoothing constraints to limit waves on the calculated curve.

The curve format chosen is the non uniform cubic B-spline. The knot values are defined by ui, and the control polygon is defined by the Pi points:

$$\overrightarrow{Q(u)} = \sum_{i=0}^{n} \vec{P}_i N_{i,m}(u) \qquad u \in [u_0, u_{n+m+1}] \qquad [1]$$

3.1. *The initial interpolation method*

The initial method is based on the interpolation of a set of points belonging to the ideal tool-path curve by a non uniform cubic B-spline. As exposed in section 2.3, a partitioning of the curve in C^2 continuous portions is necessary [DUC 98a].

Therefore, the problem consists in the calculation of the curve Q, defined by the control points Pi and the knot values ui, that verifies:

$$\vec{Q}(\mu_i) = \sum_{j = t-m}^{t} \vec{P}_j N_{j,\,m}(\mu_i) = \vec{m}_i \qquad u_t \le \mu_i \le u_{t+1} \qquad i = 0, ..., p \qquad [2]$$

where $\vec{m}_i$ are the interpolation points and µi the associated parameters.
The problem is a simple interpolation problem which is linear and easy to solve. The resolution is done in four steps:

-choice of the interpolation points on the ideal curve,
-calculation of the knot values so that the matrix is well conditioned,
-expression of the interpolation matrix and resolution,
-checking of the respect of the machining tolerance.

If the respect of the machining tolerance is not ensured, interpolation points must be added.

3.2. *Additional tangency constraints*

The use of additional tangency constraints guarantees a better continuity at the linking of curves. The influence is limited to the first and the last arcs of the considered portion. Thus, the tangent vector directions must be precisely defined from the ideal curve. The respect of these *tangency* constraints in ensured if the first curve derivatives at the extremities are parallel to the imposed directions, that is:

$$\frac{d\vec{Q}}{du}(u_0) = \frac{m}{u_4 - u_0}(\overrightarrow{P_0 P_1}) \qquad \frac{d\vec{Q}}{du}(u_n) = \frac{m}{u_{n+m+1} - u_{n+1}}(\overrightarrow{P_{n-1} P_n}) \qquad [3]$$

This problem admits a unique solution if the tangent vector is completely defined in norm and direction. This can be carried out for the abscissa is equal to the curve parameter. Then:

$$\begin{aligned} \vec{t}_0 &= \frac{3}{u_4 - u_0}(\overrightarrow{P_0 P_1}) \\ \vec{t}_n &= \frac{3}{u_{n+3+1} - u_{n+1}}(\overrightarrow{P_{n-1} P_n}) \end{aligned} \qquad [4]$$

3.3. *Smoothing constraint (SC)*

The smoothing of the curve allows the decrease in the curve oscillations. The smoothing is performed when an initial curve is calculated that respects the geometric constraints and the expected precision. Therefore, the smoothing consists in the deformation of the initial curve, that is the modification of the set of control points. The respect of the linking constraints imposes that both the first and the last control point cannot be modified.

Consider a set of m_i points, sampling of the interpolating curve, we can define a set of m'_i points by displacement of the first set of points along the normal to the curve, with a value equal to the machining tolerance. This gives a precise sampling of the positioning track of the curve (figure 2). In this way, the m'i points allows the location of the calculated curve relatively to the ideal curve. The displacement vector, $\overrightarrow{dPi}$, of each control point P_i of the curve, defines the deformation of the curve.

$$\overrightarrow{Q'(u)} = \sum_{i=0}^{n} (\vec{P_i} + \overrightarrow{dP_i}) N_{i,m}(u) \qquad u \in [u_0, u_{n+m+1}] \qquad [5]$$

The objective is to minimise (according the least-square criterion):

$$\sum_{Mi}\left(\overrightarrow{M_i} - \sum_{j=t-m}^{t} (\vec{P_i} + \overrightarrow{dP_i}) N_{j,m}(\mu_i)\right)^2 \qquad Mi = mi \cup m'i \qquad [6]$$

The smoothing of the curve permits to limit wave risks (curve a) and the calculated curve (curve b) is contained in a space located relatively to the ideal curve.

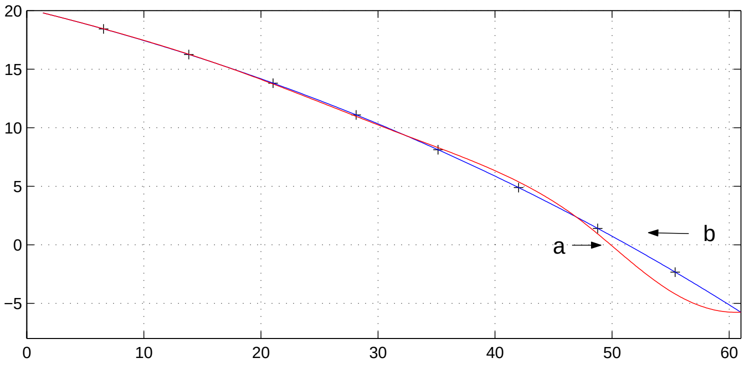


Figure 3. *Improvement using the SC, a) initial curve, b) Curve with the SC*

3.4. *The oscillation problem*

Oscillations, or waves, on the curve can be limited with the introduction of a constraint of convexity [SAP 90],[PIG 96]. That means that, for the considered portion, the tool-path is calculated so that the curve presents a unique convexity direction. In particular, this convexity constraint can be added when oscillations are detected.

Let us consider the 2D case. From the set of control points of the calculated curve respecting the machining tolerance, we consider three successive control points P_{i-1}, P_i, P_{i+1}, from i =1 to n-1. We calculate the cross product between the vectors $\overrightarrow{P_{i-1}P_i}$ and $\overrightarrow{P_iP_{i+1}}$. A change in the direction of the resulting vector point out a change in the convexity direction of the polygon. Following the variation diminishing property of the spline functions, we can conclude that the change on the polygon is the same on the curve.

When a change in the convexity direction is determined, the following optimization algorithm allows the removing the oscillation:

-let δP, be the displacement of the P control point,

-objective function: to minimise the displacement δP,

-constraints: the vectors calculated from the cross product are all of the same direction.

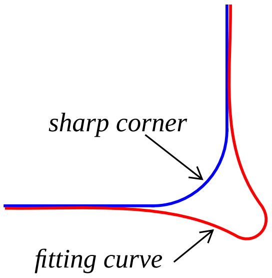


Figure 4. *deformation of a sharp corner fitting curve [KJE 83]*

However, this constraint is too strong, and the resulting curve generally no longer belong to the pipe. Tests with the convexity constraints have not been developed yet.

4. Performance assessment of the free-form curve format

First tests are performed in 2D, and the form to be machined is the curve proposed figure 5. This curve is relatively simple, but presents concavity inversion. It consists of successive arcs of circle presenting C^1 continuous linkings.

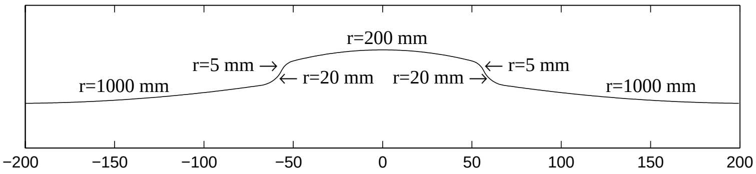


Figure 5. *Form proposed to machine*

4.1. *Procedure*

Tests are carried out with data resulting from a preliminary calculation. The previous partioning of the curve into seven portions is performed using algorithms in [DUC 98b]. Thus, the ideal tool path is defined by a set of points representing the location of the tool extremities. A set of initial points is calculated so that the maximum distance between two points is equal to 0.1 mm with a machining tolerance equal to 0.01 mm. From this point set, a subset of interpolating points, regularly scattered is extracted. When it is necessary the tangents at both extremities are also calculated.

Tool paths in a cubic B-spline format are calculated following the different methods previously exposed. The performance of each one is evaluated following the criteria :

-static criteria: the minimal and maximal deviation, the absolute value of the mean deviation, the standard deviation.

-dynamical criteria: the minimal length per points, the mean length per points, the existence of waves.

The deviation is evaluated from the distance between a sampling of the calculated tool path and the CAD model of the surface. The calculation results from a projection of the points onto the surface then onto the tool-path. The deviation represents the smallest distance minus the tool radius [DUC 00]. Note that a variation in the deviation sign does not always imply waves onto the curve. Indeed, deviations represent distances of the calculated tool-path to the ideal tool-path, and can be positive or negative for curvature radii can be different for both curves.

4.2. *Tests*

Various tests have been carried out under different interpolation conditions. Table 1 gathers all the tests and presents the different conditions. These conditions are issued from the previously exposed geometric constraints. Thus, from an initial interpolation problem with a minimal number of control points, we can consider additional interpolation points, additional tangency constraints or the smoothing of the curve (test (a) to (f)). The influence of the initial number of control points is also envisaged. Therefore, a second set of tests is carried out with an initial number of control points equal to 8 (tests (g) to (l)). Last, tests (m) and (n) concern respectively, the usual linear interpolation and the Nurbs interpolation, from a CAM software.

Table 1 : Tests

Test	**Number of initial control points**	**linking constraints**	**smoothing**
a	2	2 additional points	no
b	2	2 additional points	yes
c	2	tangency	no
d	2	tangency	yes
e	4	none	no
f	4	none	yes
g	8	2 additional points	no
h	8	2 additional points	yes
i	8	tangency	no
j	8	tangency	yes
k	8	none	no
l	8	none	yes
m	Linear Interpolation (CAM Software)		
n	Nurbs (CAM Software)		

4.3. *Test results*

Results are presented in deviation graph form (figures 6 and 7). Table 2 gathers evaluation criteria for test 1. The first significant result is the behavior of the interpolation with the smoothing constraint. Indeed, the smoothing of the curve implies the centering of the tool path within the track. Therefore, the obtained deviations are almost always positive and with a mean value equal to 0.0044mm (0.0033mm when tangency constraints are imposed), that makes the curve close to the median line of the track. This phenomenon can be seen on the deviation graph for tests (b), (d), and (f). In other cases, deviations are positive and negative (figure 6). Curves seem more accurate, unless for test (a) for which the global deviation reaches 0.0083 mm (max deviation - min deviation). Note that in figure 6, the distance between two succesive dotted lines is equal to 5 micrometers.

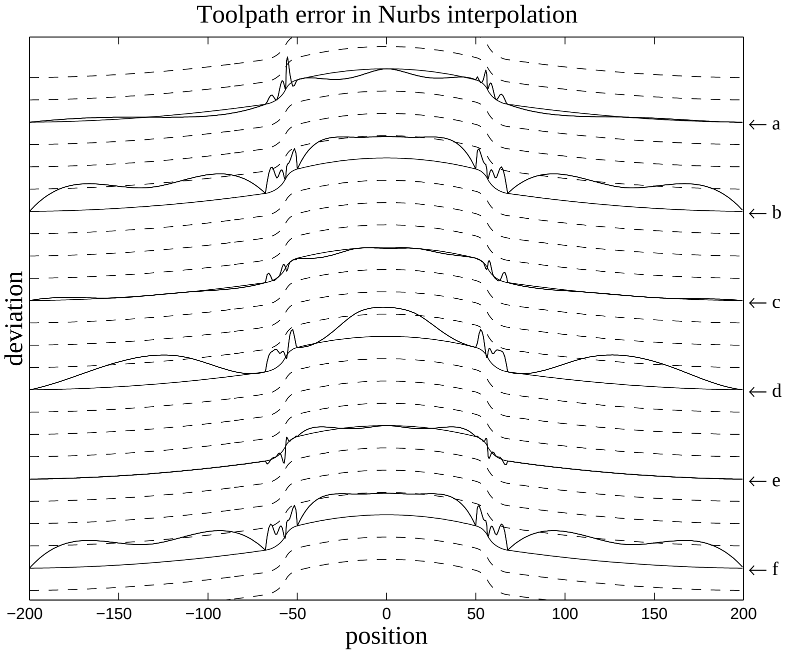


Figure 6. *Analysis of the deviations for test 1*

Let us now see, the influence of additional constraints near the linking points. The addition of points distant of 0.1 mm from both extremities involves a greater deviation than when tangency constraints are imposed, except when smoothing constraint is present (tests (a),(c) and (b),(d)). Imposing tangency constraints does not lead to a significant improvement of the deviations compared to sole interpolation (tests (c),(e) and (d),(f)). But, tangency constraints involve a good behavior at the linking points of successive B-spline curves. However, this supposes an adequate partitioning with exact tangent vectors. If not, variations may alter the behavior of the curves near extremities or near linking points.

Table 2 : Evaluation criteria for test 1

Criteria	a	b	c	d	e	f
points	35	35	35	35	35	35

Table 2 : Evaluation criteria for test 1

Criteria	a	b	c	d	e	f
min dev.(mm)	-0.0017	-2e-5	-0.0012	-0.0008	-0.0032	-0.0004
max dev. (mm)	0.0066	0.0059	0.0023	0.0065	0.0017	0.0059
$\sum \lvert dev \rvert / n$	0.0006	0.0044	0.0003	0.0033	0.0002	0.0044
standard dev.	0.001	0.0014	0.0004	0.0023	0.0004	0.0014
mini length per points	2.188	2.188	2.187	2.188	2.188	2.188
mean length per points	12.145	12.145	12.145	12.145	12.145	12.145
waves	no	no	no	no	no	no

The second set of tests is carried out with a greater number of initial control points, in order to be closer to solutions proposed by CAM systems. Results show a real improvement of the precision that can reach values less than the micrometer (figure 8). Note that the best curve is obtained for an initial number of control points equal to 8 without tangency constraints (k).

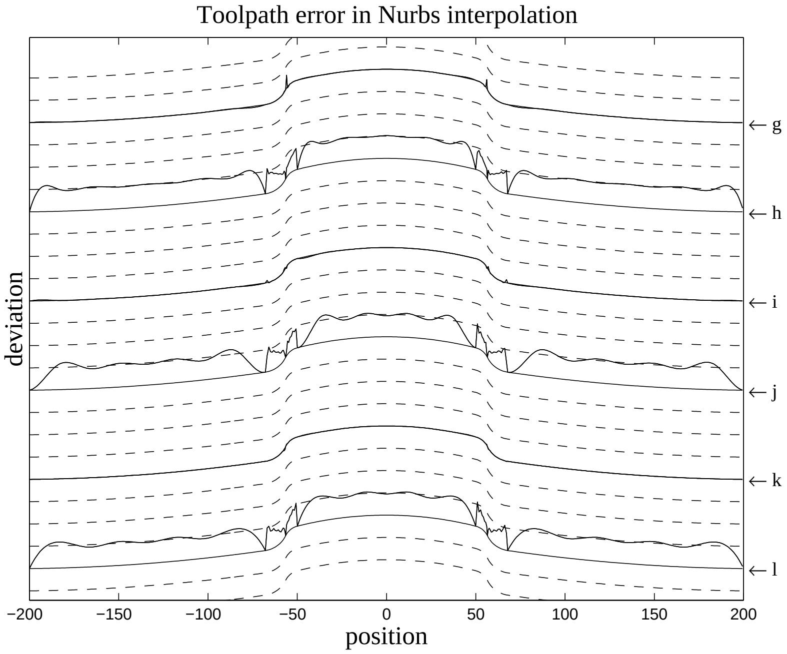


Figure 7. *Deviation graph with 8 initial control points*

4.4. *Comparison between the different interpolation methods*

The aim of this section is to evaluate the contribution of our methods relatively to those proposed by CAM systems. The comparison is done in particular, with solutions proposed by CAM systems for both the linear and the polynomial interpolation. The CAM systems compute two complete toolpaths, which respect a machining tolerance equal to 0.01 mm.

The first noting concerns the number of points which is clearly decreased with free-form curve format relatively to the linear format (table 3). Therefore, the number of points to transmit to the numerical controller is decreased by 45 to 75%, with the same precision, or better for particular portions of the tool-path.

Second, we can notice differences on precision. In particular, the tool path resulting from the CAM system, and whatever the format, presents points for which the deviation is greater than the authorized one. These deviations generally appear near the linking points of two arcs of circles (tests (m) and (n)). Although the number of points is important in this zones, the precision is not respected whereas it is the case with our methods. Here, the problem is entirely linked to a wrong partitioning of the ideal curve: a correct partitioning allows the respect of the precision with a minimum number of points.

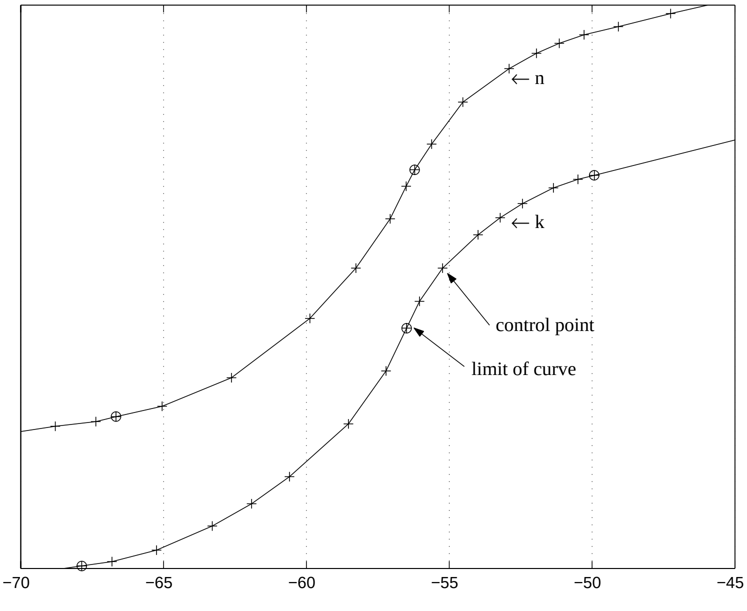


Figure 8. *Control points and partioning for the test n and k*

As the dynamical criteria are concerned, both the minimal and the mean length per points increased when using the free-form curve format. This result is very significant: the numerical controller does not require slowing down to treat all the points. However, the tool path issued from the CAM system (test (n)) presents undesirable waves. This problem also results from a wrong partitioning that does not respect the C2 discontinuity. Indeed, tool-paths obtained using our method for which a partitioning of the tool-path curve at the C2 discontinuity points is done, are wave free. As a result, the partitioning appears to be an essential step in the calculation of tool-paths in free-form curve format.

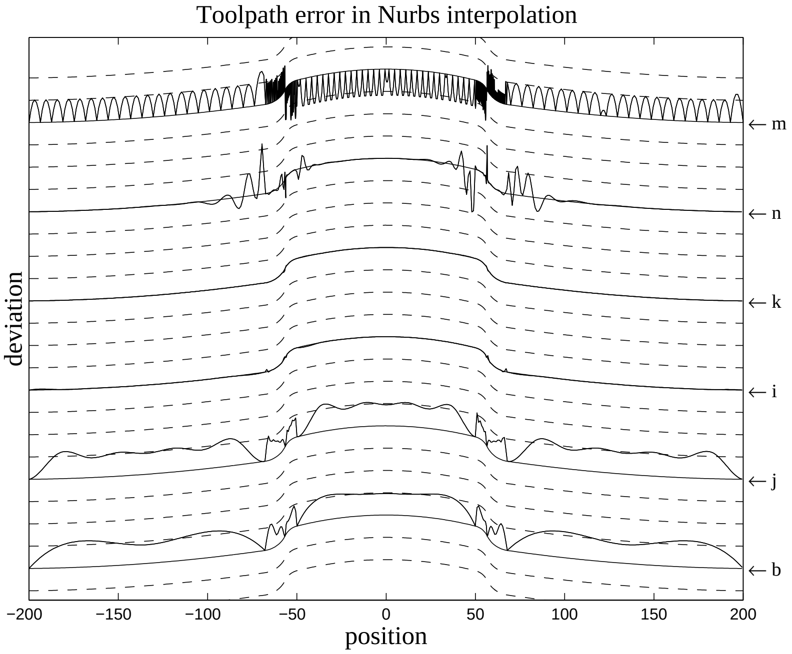


Figure 9. *Comparison between the different interpolation methods*

Table 3 : Evaluation criteria for the comparison

Criteria	**m**	**n**	**k**	**i**	**j**	**b**
points	148	82	63	77	77	35
min dev.(mm)	-0.0085	-0.0097	-0.0004	-0.0004	-2e-5	-2e-5
max dev. (mm)	0.0075	0.0113	0.0002	0.0007	0.0061	0.0059
$\sum \lvert dev \rvert / n$	0.0031	0.0005	1e-5	4e-5	0.0043	0.0044
standard dev.	0.0038	0.0015	3e-5	9e-5	0.0016	0.0014
mini length per points	0.034	2.009	1.094	0.875	0.875	2.188
mean length per points	0.531	5.098	6.660	5.433	5.433	12.145

Table 3 : Evaluation criteria for the comparison

Criteria	m	n	k	i	j	b
waves	no	yes	no	no	no	no

5. Conclusion

This paper deals with problems linked to free-form machining in high-speed milling, and the necessity to generate tool path in free-form curve format. Moreover, an analysis of the set of parameters that influence the precision and the quality of the calculated tool-path is performed.

First of all, the tool-path calculation requires an exact representation of the ideal tool-path. Indeed, errors on the calculation of the tool locations and errors on the curve length calculation affect the precision of the tool path calculation.

The second essential factor affecting the precision of the calculated tool-path concerns the partitioning of the curve. An approximate partitioning near the discontinuity points provides alterations near these linking points.

However, the addition of the smoothing of the curve involves the error values to be decreased. The smoothing of the curve also involves a location of the calculated tool path within a track that can be defined in relation to the surface convexity or in function of the expected thickness onto the part.

Additional tangency constraints improve the precision of the calculated tool-path when the tangent vector directions are well defined, and when the partitioning is well done. If these conditions are not respected, tangency constraints lead to alterations of the quality of the calculated tool-path.

For the tests carried out, we have estimated non-necessary to add the convexity constraint. Here, in fact, the appearance of waves is strongly linked to errors in the curve partitioning.

Taking into account all these remarks, and in comparison with the linear format, the B-spline format leads to the decrease of the number of points transmitted to the NC, for the same precision. Moreover, the mean size of elementary tool-path is greater. This last point, associated to information available with the B-spline format, implies a better behavior during the machining on the machine tool. In particular, the feedrate of the tool may be maximized.

To conclude, the free-form curve format seems a significant contribution to obtain a high quality level of machined surfaces. This point should be confirmed with tests of free-form machining in HSM.

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